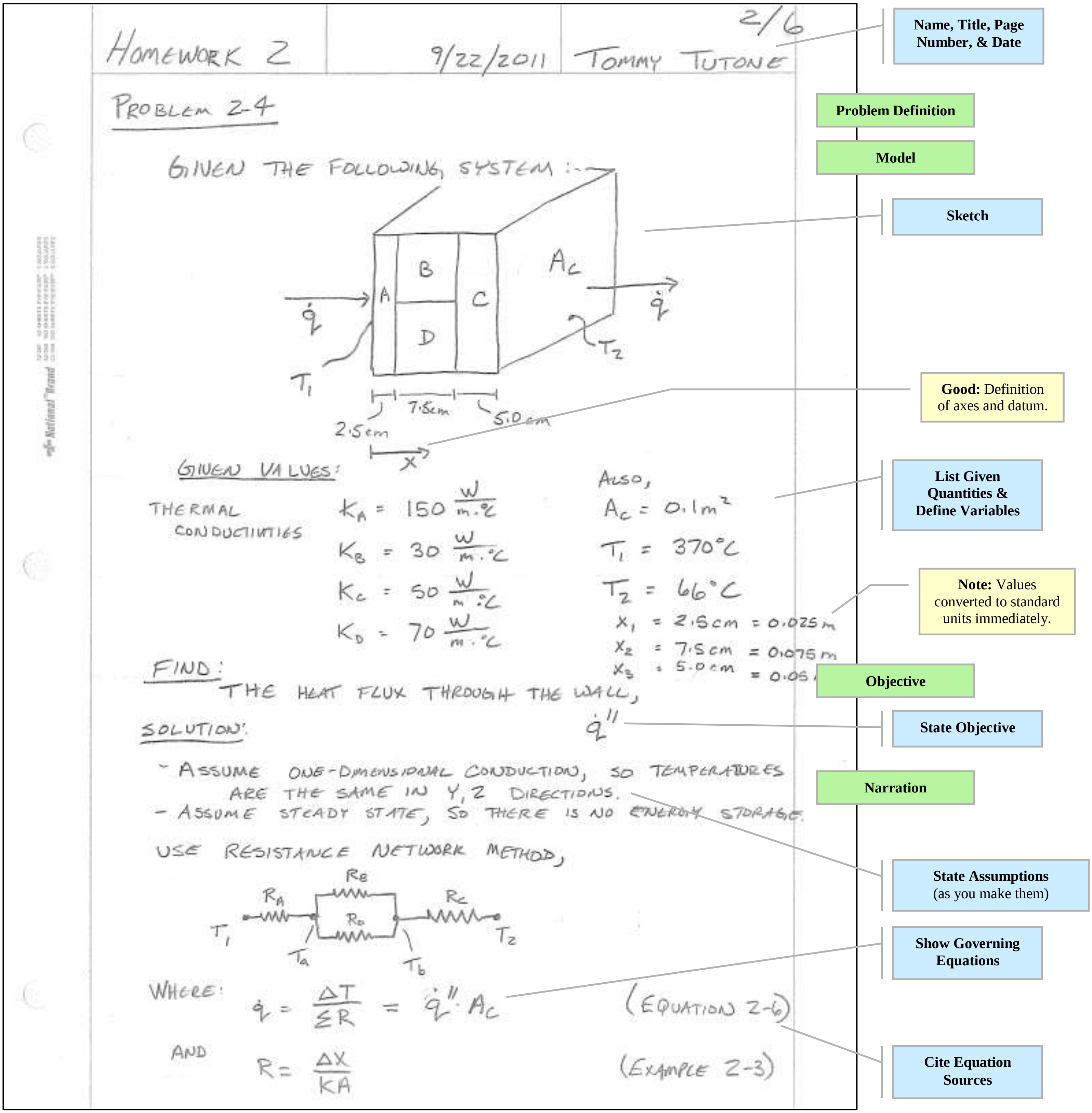
3. Annotated Example

Below is an example problem, with its components labeled and explained.



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PROBLEM 2-4

GIVEN THE FOLLOWING SYSTEM :-

GIVEN VALUES:

THERMAL CONDUCTIVITIES

$K_A = 150 \frac{W}{m \cdot °C}$

$K_B = 30 \frac{W}{m \cdot °C}$

$K_C = 50 \frac{W}{m \cdot °C}$

$K_D = 70 \frac{W}{m \cdot °C}$

ALSO,

$A_C = 0.1 m^2$

$T_1 = 370°C$

$T_2 = 66°C$

$x_1 = 2.5 cm = 0.025 m$

$x_2 = 7.5 cm = 0.075 m$

$x_3 = 5.0 cm = 0.05$

FIND:

THE HEAT FLUX THROUGH THE WALL, $\dot{q}''$

SOLUTION:

- ASSUME ONE-DIMENSIONAL CONDUCTION, SO TEMPERATURES ARE THE SAME IN Y, Z DIRECTIONS.
- ASSUME STEADY STATE, SO THERE IS NO ENERGY STORAGE.

USE RESISTANCE NETWORK METHOD,

WHERE: $\dot{q} = \frac{\Delta T}{\Sigma R} = \dot{q}'' \cdot A_C$ (EQUATION 2-6)

AND $R = \frac{\Delta X}{KA}$ (EXAMPLE 2-3)

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PROBLEM 2-4 cont   TOMMY TUTONE

THE SUM OF RESISTANCES IS:

$$\Sigma R = R_A + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} + R_C$$

WHERE SECTIONS B & D ARE COMBINED LIKE PARALLEL RESISTORS.

I'LL FURTHER ASSUME B & D HAVE EQUAL AREAS $= \frac{1}{2} A_C$.

COMBINING EQUATIONS,

$$\dot{q}'' A_C = \frac{T_1 - T_2}{\frac{x_1}{K_A A_C} + \frac{1}{\frac{1}{\frac{2x_2}{K_B A_C}} + \frac{1}{\frac{2x_2}{K_D A_C}}} + \frac{x_3}{K_C A_C}}$$

CANCELLING + SIMPLIFYING,

$$\dot{q}'' = \frac{T_1 - T_2}{\frac{x_1}{K_A} + \frac{2x_2}{K_B + K_D} + \frac{x_3}{K_C}}$$

SUBSTITUTING VALUES,

$$\dot{q}'' = \frac{370°C - 66°C}{\frac{0.025\,m}{150\,\frac{W}{m°C}} + \frac{2(0.075\,m)}{(30+70)\,\frac{W}{m°C}} + \frac{0.05\,m}{50\,\frac{W}{m°C}}}$$

$$= 113900\ \frac{W}{m^2}$$

$$\boxed{\dot{q}'' = 114\ \frac{kW}{m^2}}$$

Note: Use small sentences to narrate your solution.

Good: Description of your logic for the equations you have chosen.

Note: Assumptions may be made and shown as you go, not just at the beginning.

Solve

Substitute Values

Show Units!

NOTE: Including units has kept you from using x in [cm] instead of [m].

Indicate Final Answer

Evaluate

Evaluate your work: After you have solved a problem, it is very important to ask yourself, "Does this make sense?" This is expected of you, but often you do not show it on paper. At the minimum, it will help you find errors in your solution.

For example perform an order-of-magnitude comparison:

"This flux value is very high. A toaster consumes ~1 kW of energy –or— the solar constant on earth is ~1 kW/m^2. This value is two orders of magnitude larger. However, the large ΔT, small Δx, and large k values are all extreme and justify this flux value."

Or, perform a unit analysis by cancelling the units from your substituted values. Should energy be represented by meters?

Review!